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Evolution of switch-on and switch-off shocks in a gas of finite electrical conductivity

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The evolution of plane switch-on and switch-off shocks in the presence of small normal disturbances is examined. (Normal disturbances are those in which the perturbed quantities are functions only of time and the distance from the plane of the shock wave.) In the situation under discussion the magnetic diffusivity of the ambient gas is much greater than each of the viscous diffusivities and the thermal diffusivity.

It is shown that small normal disturbances can eventually cause appreciable changes in the flow pattern around either switch-on or switch-off shocks. Numerical computations are carried out for both a specific null switch-on and a specific null switch-off shock, in the presence of a known disturbance. The results of each of these computations, for large times, are in agreement with analytic, asymptotic solutions which are obtained.

A mechanism is suggested whereby switch-on and switch-off shocks, across which an appreciable deflexion of the direction of the magnetic field lines takes place, can adjust themselves to small normal disturbances.

1. Introduction

The stability of plane, oblique magneto-gasdynamic shocks to small normal disturbances has been considered by Akhiezer, Liubarskii & Polovin (1958) and Syrovatskii (1959), the ambient gas being treated as ideal and perfectly conducting. (Thus the shock was treated as a discontinuity.) They found that switch-on and switch-off shocks cannot emit enough small-amplitude plane waves to adjust themselves to small normal disturbances. Anderson (1963) also came to the same decision. They all concluded that in the presence of small disturbances switch-on and switch-off must 'spontaneously' disintegrate into some configuration of stable plane waves.

This paper discusses the above initial value problem for cases in which the magnetic diffusivity of the ambient gas is much greater than any other diffusivity. (This is a realistic situation.) Consequently, in contrast to previous works, there is now a natural length scale (namely the shock thickness) available. Moreover, for this problem continuum theory can be valid.

A cartesian set of axes $Oxyz$ is employed. The x-axis points downstream, in the direction of variation. This is normal to the shock plane and is also referred to as the longitudinal direction. The Oy-axis is chosen so that the magnetic field (in the undisturbed state) lies in the (x, y)-plane. Oz is termed the transverse direction. Furthermore, the origin of co-ordinates O is chosen moving (uniformly) within the shock in such a way that **V** is parallel to **B** ahead and behind the front (if $B_x \neq 0$, this can always be done; see Shercliff 1960).

It should be noted that once we start to work out problems in which the region of dissipation is not of negligible width, we shall have to define the longitudinal position (in the shock) of the origin of co-ordinates.

Todd (1964) and Todd (1965) cover the same situation for normal and oblique magneto-gasdynamic shocks.

2. Steady-state shock structure

In switch-on and switch-off shocks,

$$G = \rho V_x = \text{const.}, \tag{1}$$

$$F_x = p + \rho V_x^2 + (B_y^2/2\mu) = \text{const.}, \tag{2}$$

$$H = e + (p/\rho) + \tfrac{1}{2}(V_x^2 + V_y^2) = \text{const.}, \tag{3}$$

$$F_q = GV_q - (B_x B_q/\mu) = 0, \quad q = y, z, \tag{4}$$

$$V_x B_q - B_x V_q - \lambda \, dB_q/dx = 0, \quad q = y, z, \tag{5}$$

where e is the internal energy of the gas, and the other symbols have their usual meaning.

It follows from (4) and (5) that V_z and B_z are zero *throughout* the shock (this was assumed in equations (2) and (3)). Equations (4) and (5), with $q = y$, give

$$\lambda \frac{dB_y}{dx} + (m^{-2} - 1) G\tau B_y = 0, \tag{6}$$

where $\tau = \rho^{-1}$ is the specific volume, and $m = \{V_x/\sqrt{(B_x^2/\mu\rho)}\}$.

Once the equation of state is specified, equations (1) to (4) can be combined to give a null curve in the form $N(B_y, \tau) = 0$. For a reasonably well behaved gas, the correct qualitative features of switch-on and switch-off shock are given by setting $p = a^2\rho$, where 'a' is an absolute constant and neglecting the energy equation (see figure 1). A more complete discussion is given by Ludford (1959) and Anderson (1963).

There is a gasdynamic discontinuity (CD or $C'D'$) in the 'tail' of a switch-on shock if $V_x > \sqrt{\{(\partial p/\partial \rho)_s\}}$ upstream, and if $V_x < \sqrt{\{(\partial p/\partial \rho)_s\}}$ downstream. If these conditions on V_x are satisfied by a switch-off shock then that shock has a gasdynamic discontinuity in its 'nose' (e.g. CD or $C'D'$). If these conditions† on V_x are not satisfied then the switch-on or switch-off shock under consideration does not contain a subshock. *The above statements are true whatever assumptions we make about the thermodynamic behaviour of the gas.* A gasdynamic discontinuity or subshock is contained in a very thin region, relative to the overall width of the magneto-gasdynamic shock, and it runs on viscous and thermal dissipative processes. Its overall effect is exactly that of an ordinary gasdynamic shock', i.e. the Rankine–Hugoniot equations are valid across it. However such a subshock would consist of a yet thinner region, in which sharp changes in the

† A switch-on shock always satisfies the former condition. All switch-off shocks satisfy the latter condition.

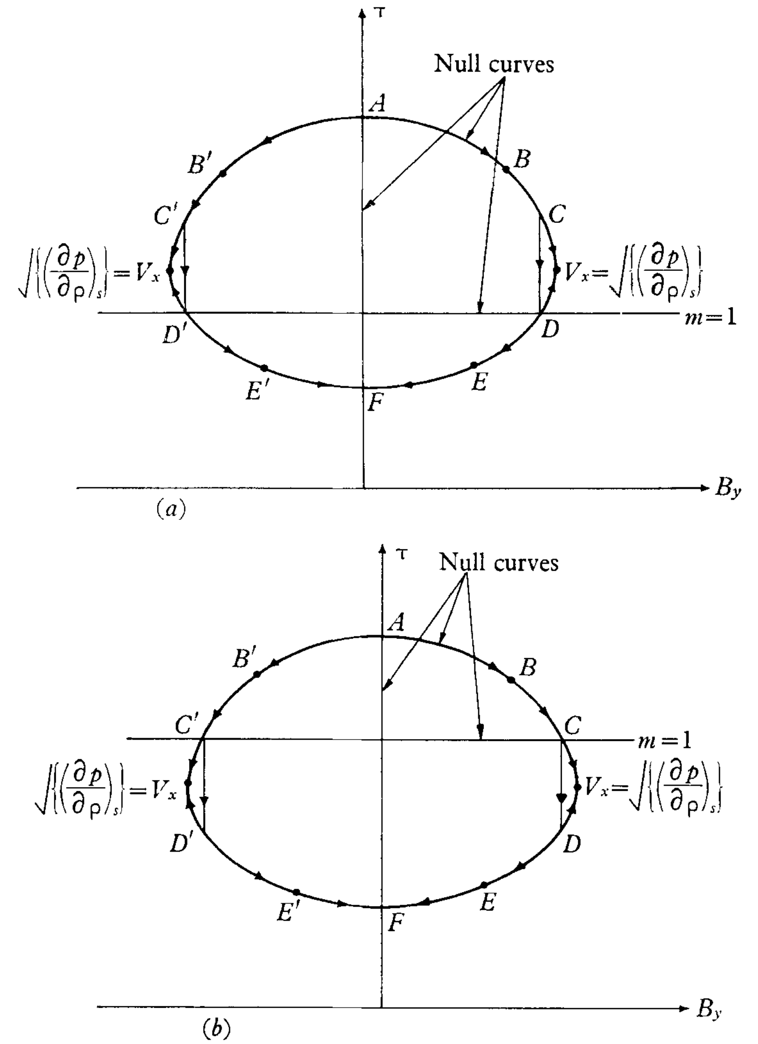

FIGURE 1. The null curves for given B_x, G, F_x. (The arrows indicate how B_y and τ are changing, as we move *downstream* through the shock region.) (*a*) *ABCD* (or *A'B'C'D'*), switch-on; *DEF* (or *D'E'F'*), switch-off. (*b*) *ABC* (or *A'B'C'*), switch-on; *CDEF* (or *C'D'E'F'*), switch-off.

properties of the ion gas take place, surrounded by a broader region over which the electron-gas regains thermodynamic equilibrium with the ion gas. For the purpose of this paper we can and will treat subshocks as discontinuities in the flow, across which mass flux, momentum flux and energy flux are conserved.

3. Discussion of the stability problem

This is given in Todd (1965), §§2.1 and 3.1. (However, the left-hand sides of equations (33) and (39) of that paper should contain a term $(-\dot{\epsilon}B^2/2\mu)$.) We shall summarize the most important points.

If the switch-on (or switch-off) shock contains a gasdynamic discontinuity we take our origin fixed at this discontinuity. If the shock does not contain a subshock, we can, *for example*, locate our axes at the point where B_y is one half of the value in the undisturbed downstream (or upstream) region.

Switch-on or switch-off shocks, across which an appreciable deflexion in the direction of the magnetic field lines takes place, are stable to disturbances in B_y, V_y, p, ρ and V_x. The degenerate types are discussed in §§5 and 7.

The remainder of this paper is devoted to small normal disturbances in V_z and B_z. The relevant equations and boundary conditions are given in Todd (1965), §2·1. The initial disturbance which we shall consider is

$$b = (B_z/B_x) = \begin{Bmatrix} f_1(x < 0) \\ f_2(x > 0) \end{Bmatrix} \quad \text{and} \quad (V_z/V_x) = \begin{Bmatrix} g_1(x < 0) \\ g_2(x > 0) \end{Bmatrix}, \tag{7}$$

where f_1, f_2, g_1 and g_2 are absolute constants.

4. Evolution of the switch-on shock

In §3 we discussed the longitudinal positioning of our set of cartesian axes. If the switch-on shock under consideration does not contain a subshock, we take $x = 0$ to be the point where $m = 0{\cdot}99\ldots 99$. If it does contain a subshock, then $x = 0$ is the position of the subshock. (It will be remembered from §2 that the subshock is situated at the downstream end of the switch-on shock.)

Thus, for the purpose of our analysis, we can take $m = 1$, for $x > 0$, and $m > 1$, for $x < 0$.

We shall seek asymptotic solutions, valid at large times, of the form

$$b = T^{\frac{1}{2}}F_1(x) + F_2(x) + O(T^{-\frac{1}{2}}) \quad \text{for} \quad x \leqslant 0, \tag{8}$$

and

$$b = f_2 + b_r + T^{\frac{1}{2}}G_1(\theta) + G_2(\theta) + O(T^{-\frac{1}{2}}) \quad \text{for} \quad T^{\frac{1}{2}} \gg \theta \geqslant 0, \tag{9}$$

where $T = (V_x^2 t/\lambda)_{+\infty}$ and $\theta = x/\sqrt{(\lambda t)}$. b_r is a constant. Note that for $\theta > 0$, V_x and λ are constant in value.

We shall require that, as $x \to -\infty$, $F_1(x) \to 0$ and $F_2(x) \to f_1$. Also, as $\theta \to +\infty$, with $(\theta/\sqrt{T}) \ll 1$, $G_1(\theta)$ and $G_2(\theta)$ and their derivatives with respect to θ all tend to zero.

In seeking asymptotic solutions of the above forms, it is assumed that the switch-on shock emits downstream a diffusing, step Alfvén wave, across which the change in b is b_r. The change in V_z across such a wave is $-V_x b_r$.

4.1. *The region $x < 0$*

Equations (1) and (2) of Todd (1965) may be combined to produce the differential equation satisfied by b. Terms of order $T^{-\frac{1}{2}}$, and higher, are neglected. The coefficients of $T^{\frac{1}{2}}$ and unity are set equal to zero. In this manner, we find that

$$\left\{\lambda \frac{d}{dx} + V_x(m^{-2} - 1)\right\} F_1(x) = 0, \tag{10}$$

and

$$\left\{\lambda \frac{d}{dx} + V_x(m^{-2} - 1)\right\} F_2(x) = -\left(\frac{z-1}{z}\right) f_1 V_x(-\infty). \tag{11}$$

The fact that $V_x m^{-2}$ is independent of x has been used in deriving the above pair of equations. z is the density ratio across the switch-on shock. $1 < z = (m^2)_{-\infty}$. Thus

$$F_1(x) = A_1 \Phi(x), \tag{12}$$

where

$$\Phi(x) = \exp\left\{-\int_x^0 \frac{V_x}{\lambda}(1-m^{-2})\,dx\right\}, \tag{13}$$

and A_1 is a constant of integration. Also

$$F_2 = A_2\Phi(x) + \left(\frac{z-1}{z}\right) f_1\{V_x(-\infty)\}\int_x^0 \{\lambda(\eta)\,\Phi(\eta)\}^{-1}\,d\eta\,\Phi(x), \tag{14}$$

where A_2 is a constant. V_z is given by

$$V_z = g_1\{V_x(-\infty)\} + V_x m^{-2}(T^{\frac{1}{2}}F_1(x) + F_2(x) - f_1) + O(T^{-\frac{1}{2}}). \tag{15}$$

4.2. *The region* $T^{\frac{1}{2}} \gg \theta > 0$

This is the region downstream of the switch-on shock. The differential equation satisfied by b is

$$\left(\frac{\partial^2}{\partial T^2} + 2\frac{\partial^2}{\partial T\,\partial X} - \frac{\partial^3}{\partial T\,\partial X^2} - \frac{\partial^3}{\partial X^3}\right) b = 0, \tag{16}$$

where $X = (V_x x/\lambda)$. Thus we have that

$$(G_1'''' + \theta G_1'')\,T^{-1} - \tfrac{1}{4}T^{-\frac{3}{2}}\{(2-\theta^2)\,G_1'' + \theta G_1' - G_1 + 4G_2'''' + 4\theta G_2'' + 4G_2'\} + O(T^{-2}) = 0,$$

where primes denote differentiation with respect to θ. Hence

$$G_1'''' + \theta G_1'' = 0, \tag{17}$$

and

$$\frac{d}{d\theta}(G_2''' + \theta G_2') = \tfrac{1}{4}\{(2-\theta^2)\,G_1'' + \theta G_1' - G_1\}. \tag{18}$$

Equation (17), together with the conditions on the behaviour of $G_1(\theta)$, for $T^{\frac{1}{2}} \gg \theta \gg 1$, gives

$$G_1(\theta) = C_1\{\exp(-\tfrac{1}{2}\theta^2) - \surd(\tfrac{1}{2}\pi)\,\theta\,\mathrm{erfc}\,(\theta/\surd 2)\}, \tag{19}$$

where C_1 is a constant, and

$$\mathrm{erfc}\,(\alpha) = 1 - \frac{2}{\surd\pi}\int_0^\alpha e^{-\eta^2}\,d\eta.$$

It subsequently follows that

$$G_2(\theta) = \tfrac{1}{8}C_1\theta\exp(-\tfrac{1}{2}\theta^2) + C_2\,\mathrm{erfc}\,(\theta/\surd 2), \tag{20}$$

where C_2 is a constant.

Equations (1) and (2) of Todd (1965), together with the condition that $V_z \to (g_2 - b_r)\,V_x$ as $\theta \to \infty$ (with $\theta/\surd T \ll 1$), give

$$(V_z/V_x) = (g_2 - b_r) + G_1(\theta)\,T^{\frac{1}{2}} + C_1\{\tfrac{1}{8}\theta\exp(-\tfrac{1}{2}\theta^2) + \surd(\tfrac{1}{8}\pi)\,\mathrm{erfc}\,(\theta/\surd 2)\} + C_2\,\mathrm{erfc}\,(\theta/\surd 2) + O(T^{-\frac{1}{2}}). \tag{21}$$

The constants A_1, A_2, C_1, C_2, and b_r are evaluated by satisfying, to order unity, the boundary conditions at $x = 0$, i.e. terms of order $T^{-(\frac{1}{2})}$ and higher are neglected. The results obtained are

$$b_r = \tfrac{1}{2}\{\tfrac{1}{2}(z+1)f_1 - f_2 + g_2 - zg_1\}, \tag{22}$$

$$A_2 = C_2 + \tfrac{1}{2}\{g_2 - zg_1 + f_2 + \tfrac{1}{2}(z+1)f_1\}, \tag{23}$$

and

$$A_1 = C_1 = \surd(2/\pi)\,(z-1)\,f_1. \tag{24}$$

It is not surprising that C_1 is independent of f_2, g_2 and g_1. Disturbances in the region $x > 0$ (f_2 and g_2) are not propagated towards the shock. Also, according to infinite conductivity theory (Akhiezer *et al.* 1958), a switch-on shock can adjust to upstream disturbances in V_z (g_1) by emitting its one outgoing Alfvén wave.

The constant C_2 remains indeterminate. It can be found by including higher-order terms in the analysis. However, all the interesting features of the flow have already been found so we shall not proceed to evaluate C_2. If we had used infinite conductivity theory (Akhiezer *et al.* 1958) to calculate b_r for a super-Alfvénic shock and then let $(m^2)_{+\infty} \to 1$, from above, we would have obtained equation (22). Also if we had used the result of Todd (1965), equation (26) of which gives b_r for a trans-Alfvénic shock, and let $(m^2)_{+\infty} \to 1$, from below, we should have obtained equation (22).

5. Evolution of null switch-on shocks

The upstream region is separated from the downstream region by a gasdynamic discontinuity, i.e. this degenerate species of switch-on shock is an ordinary gasdynamic one which has $m = 1$ on the downstream side. Consequently, the coefficients of the derivatives in equations (10) and (11) are constants.

In the undisturbed state, $B_y = 0$ on both sides of the shock and disturbances in B_y and V_y are decoupled from those in V_x, p and ρ. The shock is stable to perturbations in V_x, p and ρ, since it is supersonic upsteam and subsonic downstream. Also, for a null switch-on shock there is no specific direction for Oy other than that it lies in the plane of the shock wave. The governing equations and boundary conditions for small normal disturbances in B_y and V_y are identical with those for normal perturbations in B_z and V_z. Thus we will simply discuss disturbances in B_q and V_q, where q can represent y or z.

The disturbance chosen was of the form

$$\left.\begin{aligned} b = (B_q/B_x) = \Delta\,(\text{const.}) \quad \text{for all} \quad x \\ V_q = 0 \quad \text{for all} \quad z \end{aligned}\right\} \quad \text{at} \quad t = 0.$$

A shock density ratio (z) of two was chosen and $(\lambda_{-\infty}/\lambda_{+\infty})$ was set equal to four. We define $X = (V_x z/\lambda)$. (See also equation (16).)

Since the coefficients of the partial derivatives in the governing equations are constants, the problem can be solved exactly by using the Laplace transform. The inversions of the transform were carried out by numerical computation, in the manner described in Todd (1964). The resultant profiles of $(b/\Delta)-1$ are drawn for several values of T in figure 2. In figure 3, the value of $\{(b/\Delta)-1\}$ at $x = 0$ is plotted against $T^{\frac{1}{2}}$. The analysis of §4, with $f_1 = f_2 = \Delta$ and $g_1 = g_2 = 0$, claims to give the asymptotic behaviour of b for large values of T.

Figure 3 shows that the rate of rise of b, at the gasdynamic discontinuity, with the square root of time, is becoming linear at large values of T. The results of the previous section give $C_1 = (0{\cdot}787)\,\Delta$ for this problem. This appears to agree with the computational results.

For this problem, the analysis of §4 predicts that $b_r = (0{\cdot}25)\,\Delta$. This agrees with that which is 'observed'.

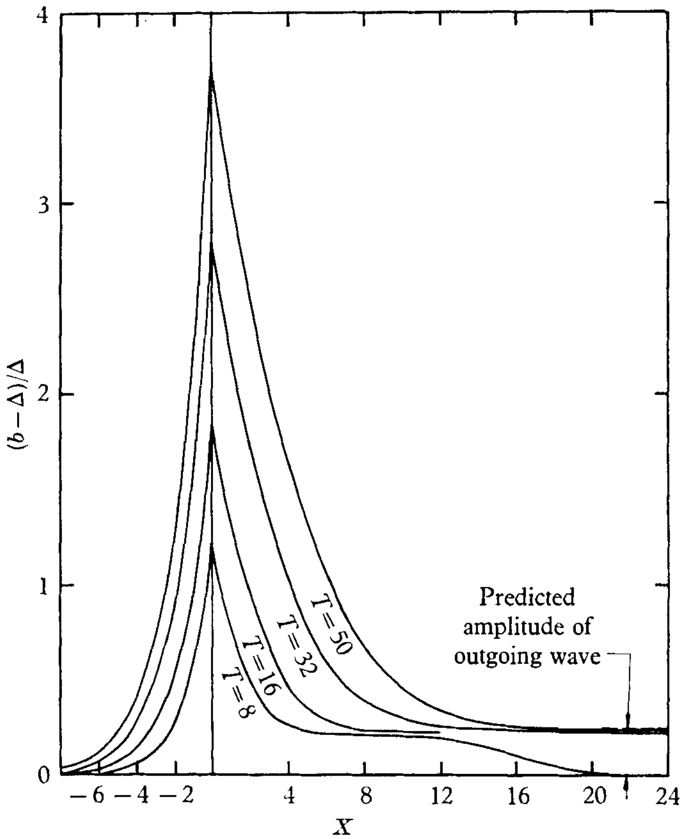

FIGURE 2. Relative increase in transverse field near a null switch-on shock.

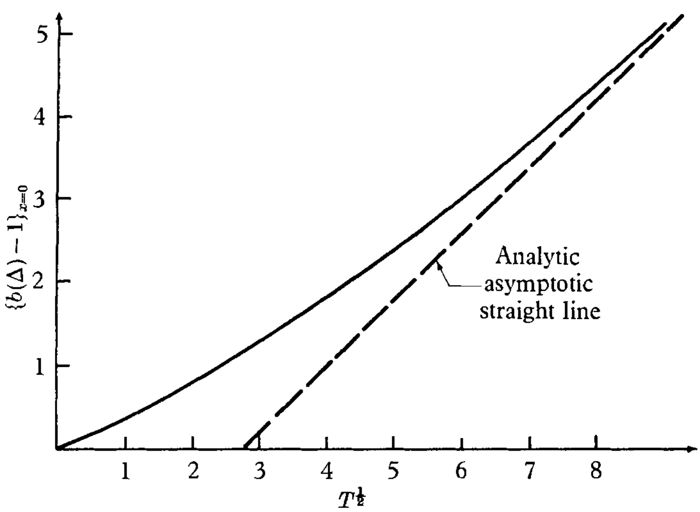

FIGURE 3. The variation of the relative increase in transverse field. at the gasdynamic discontinuity, with the square root of time. (Null switch-on case.)

A comparison between the asymptotic profiles obtained analytically (§4) and the computations is given in table 1. A value† of C_2 (namely, $-2\cdot41\Delta$ was calculated on an empirical basis, from a selection of the computational results.

T \ X	−8	−4	−2	−1	0	1	2	4	8	12
32	0·02 (0·04)	0·28 (0·31)	0·89 (0·84)	1·58 (1·39)	2·79 (2·29)	2·20 (1·73)	2·71 (1·28)	1·01 (0·73)	0·39 (0·32)	0·25 (0·16)
50	0·04 (0·06)	0·42 (0·46)	1·25 (1·25)	2·16 (2·06)	3·71 (3·40)	3·06 (2·76)	2·50 (2·22)	1·64 (1·41)	0·68 (0·59)	0·34 (0·33)

TABLE 1. Some comparison between the analytic, asymptotic profiles, obtained for large times, and the exact shock profiles of the relative transverse field. (The figures corresponding to the analytic, asymptotic profiles are shown in parentheses.)

The discrepancies between the two sets of results in table 1 are of the order of the error which is to be expected for the moderate times under consideration.

6. Evolution of the switch-off shock

Some switch-off shocks contain gasdynamic discontinuities at the upstream end of the region of ohmic dissipation. We would choose $x = 0$ to coincide with the position of the subshock. However, if this gasdynamic discontinuity was absent, we would choose $x = 0$ to be the point at which $m = 0\cdot999\ldots9$. Thus, for the purpose of our analysis, we may take $m = 1$ for $x < 0$, $m < 1$ for $x > 0$.

We shall seek asymptotic solutions, valid at large times, of the form

$$b = f_1 + T^{\frac{1}{2}}G_1(\theta) + G_2(\theta) + O(T^{-\frac{1}{2}}), \quad T^{\frac{1}{2}} \gg -\theta \geqslant 0, \tag{25}$$

where $\quad T = t(V_x^2/\lambda)_{+\infty} \quad$ and $\quad \theta = \{x/\sqrt{(t\lambda_{-\infty})}\}/\sqrt{Q},$

$Q = (V_x^2/\lambda)_{+\infty}/(V_x^2/\lambda)_{-\infty}$. $G_1, dG_1/d\theta, G_2, dG_2/d\theta$, etc., all tend to zero as $\theta \to -\infty$, with $(-\theta/\sqrt{T}) \ll 1$.

$$b = T^{\frac{1}{2}}F_1(x) + F_2(x) + O(T^{-\frac{1}{2}}) \quad \text{for} \quad x \geqslant 0. \tag{26}$$

We shall assume that the switch-off shock emits downstream a diffusing, step Alfvén wave, across which the change in b is b_r. The change in V_z across such a wave is $(-\sqrt{z}b_r u_{+\infty})$.

The analysis of this section is completely analogous to that of §4. Thus we shall simply present the results.

6.1. *The region $x \geqslant 0$*

For this region

$$b = (A_2 + A_1 T^{\frac{1}{2}})\,\Phi(x) + V_x(+\infty)\,(z-1)\,(f_2 + b_r)\int_0^x \{\lambda(\eta)\,\Phi(\eta)\}^{-1} d\eta\,\Phi(x) + O(T^{-\frac{1}{2}}), \tag{27}$$

where A_1 and A_2 are constants of integration, and

$$V_z = (g_2 - z^{\frac{1}{2}}b_r)\,u_{+\infty} + V_x m^{-2}(b - f_2 - b_r) + O(T^{-\frac{1}{2}}). \tag{28}$$

† See remarks at the end of §4.

6.2. *The region* $T^{\frac{1}{2}} \gg -\theta \geqslant 0$

For this region

$$G_1(\theta) = C_1[\exp(-\tfrac{1}{2}Q\theta^2) + \sqrt{(\tfrac{1}{2}\pi Q)}\,\theta\,\mathrm{erfc}\{-\theta\sqrt{(\tfrac{1}{2}Q)}\}], \tag{29}$$

and

$$G_2(\theta) = C_2\,\mathrm{erfc}\{-\theta\sqrt{(\tfrac{1}{2}Q)}\} + \tfrac{1}{8}C_1[-\theta Q\exp(-\tfrac{1}{2}Q\theta^2) + \sqrt{(\tfrac{1}{2}\pi Q)} \times \mathrm{erfc}\{-\theta\sqrt{(\tfrac{1}{2}Q)}\}], \tag{30}$$

$$(V_z/V_x) = g_1 + T^{\frac{1}{2}}G_1(\theta) + C_2\,\mathrm{erfc}\{-\theta\sqrt{(\tfrac{1}{2}Q)}\} - \tfrac{1}{8}C_1[\theta Q\exp(-\tfrac{1}{2}Q\theta^2) + 3\sqrt{(\tfrac{1}{2}\pi Q)}\,\mathrm{erfc}\{-\theta\sqrt{(\tfrac{1}{2}Q)}\}] + O(T^{-\frac{1}{2}}). \tag{31}$$

Once again the constants A_1, A_2, C_1, C_2 and b_r are evaluated by satisfying the boundary conditions at $x = 0$ to order unity, i.e. terms of order $T^{-\frac{1}{2}}$ and higher are neglected. It is found that

$$b_r = (\sqrt{z}+1)^{-2}\{2zf_1 - (z+1)f_2 + 2g_2 - 2zg_1\}, \tag{32}$$

$$A_1 = C_1 = \left(\frac{z-1}{z}\right)\left(\frac{2}{\pi Q}\right)^{\frac{1}{2}}(f_2 + b_r), \tag{33}$$

and

$$A_2 = C_2 + f_1 + \frac{(z-1)}{8z}(f_2 + b_r). \tag{34}$$

C_2 remains an unknown constant. It could be evaluated by including higher-order terms in the analysis.

If we had used infinite-conductivity theory (Akhiezer *et al.* 1958) to calculate b_r for a sub-Alfvénic shock and then let $(m^2)_{-\infty} \to 1$ *from below* we could obtain equation (32). Also, if we take the result of Todd (1965), which gives b_r for a trans-Alfvénic shock, and let $(m^2)_{-\infty} \to 1$, from above, we would once again obtain equation (32). It is not surprising that C_1 (the rate of rise of (B_z/B_x), at $x = 0$, with $T^{\frac{1}{2}}$) depends only on $(b_2 + b_r)$. According to infinite conductivity theory (Akhiezer *et al.* 1958) a switch-off shock can only adjust to those disturbances for which $(b)_{\text{downstream}}$ equals zero, after the passage of the one outgoing Alfvén wave. (For disturbances in this latter category, the shock would simply change to a slightly different switch-off shock.)

7. Evolution of the null switch-off shocks

A null switch-off shock is a gasdynamic discontinuity which has $m = 1$ on the upstream side. Just as for the null switch-on shock (see beginning of §5) the analysis for disturbances in B_z and V_z can be applied to disturbances in B_y and V_y. Thus we shall treat $b = (B_q/B_x)$ and V_q, where q can be y or z.

The specific disturbance chosen was of the form

$$\left.\begin{aligned} b &= \Delta(\text{const.}) \quad \text{for all } x \\ V_q &= 0 \qquad\qquad\quad \text{for all } x \end{aligned}\right\} \text{ at } t = 0,$$

i.e.

$$f_1 = f_2 = \Delta \quad \text{and} \quad g_1 = g_2 = 0.$$

The null switch-off shock under consideration is characterized by $z = 2$ and $Q = 1$. Once again (see §5) the coefficients of the partial derivatives in the governing equations are constants and the Laplace transform can be used to solve the problem exactly. The inversions of the transform were carried out by numerical computations, in a manner identical to that described in Todd (1964) for the trans-Alfvénic normal shock.

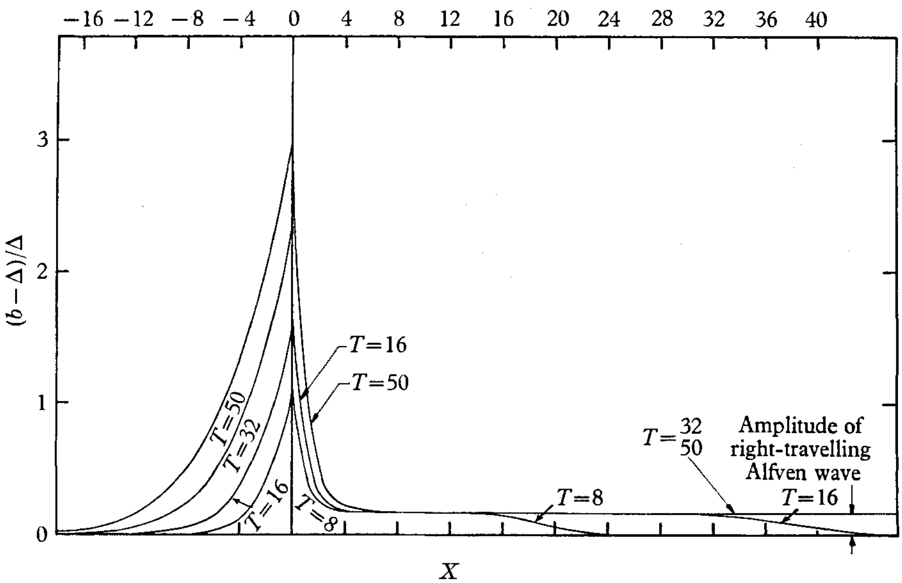

FIGURE 4. Increase in relative transverse field near a null switch-off shock.

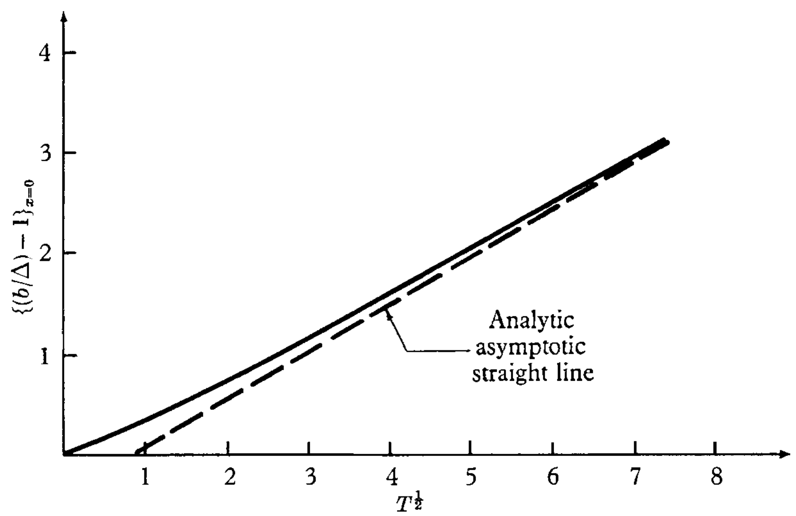

FIGURE 5. The variation of the relative increase in transverse field, at the gasdynamic discontinuity, with the square root of time. (Null switch-off case.)

The resultant profiles of $\{(b/\Delta)-1\}$ are drawn in figure 4, for several values of T. In figure 5, the value of $\{(b/\Delta)-1\}$ at $x=0$ is plotted against $T^{\frac{1}{2}}$. The analysis of § 6 with $f_1 = f_2 = \Delta$, $g_1 = g_2 = 0$, $z = 0$ and $Q = 1$, claims to give the asymptotic behaviour of b for large values of T. Figure 5 shows that the rate of rise of b, at the gasdynamic discontinuity, with the square root of time, is becoming linear at large values of T. The results of the previous section give this rate of rise as $(0\cdot467)\,\Delta$. This appears to agree with the computational results.

The analysis of § 6 also predicts that $b_r = (0\cdot172)\,\Delta$. This agrees with that which is 'observed'.

T \ X	−12	−8	−4	−2	−1	0	1	2	4	8
32	0·04	0·22	0·83	1·44	1·84	2·33	0·92	0·43	0·20	0·17
	(0·04)	(0·20)	(0·78)	(1·36)	(1·77)	(2·24)	(0·94)	(0·45)	(0·21)	(0·17)
50	0·15	0·49	1·33	2·03	2·47	2·98	1·17	0·52	0·22	0·17
	(0·14)	(0·47)	(1·28)	(1·97)	(2·41)	(2·91)	(1·18)	(0·54)	(0·22)	(0·17)

TABLE 2. Some comparison between the analytic, asymptotic profiles obtained for large times and the exact shock profiles of the relative transverse field. (The figures corresponding to the analytic, asymptotic profiles are shown in parentheses.)

A comparison between the asymptotic profiles predicted by the analysis of § 6 and the computations is given in table 2. A value† of C_2 (namely, $-0\cdot47\Delta$) was calculated, on an empirical basis, from a selection of the computational results.

The discrepancies between the two sets of results given in table 2 are well within that which could be expected for the moderate times under consideration.

8. A final breakdown configuration

§ 5 of Todd (1965) discusses the eventual fate of strong‡ switch-on and switch-of shocks which are subjected to our type of disturbance. Figures 6 and 7 illustrate these proposals for a switch-on case.

C. K. Chu & R. Taussig (1965, private communication) have carried out numerical computations of the whole non-linear development. They find precisely what is outlined in Todd (1965). However, a pseudo-viscosity factor is used in their finite difference scheme.

9. Conclusions

It is clearly shown that, if small disturbances, with a length scale large compared to the shock thickness, are incident upon a switch-on or switch-off shock, large changes in the flow pattern will be created *in the vicinity* of the original shock. In the discussion of § 8, the author shows that these changes are consistent with having something very close to an Alfvén simple wave in juxtaposition with a shock which is very similar to the original. (The substance of the latter sentence does not apply to cases in which only a very small deflexion in the direction of the magnetic field lines takes place across the original switch-on switch-off shock.)

† See remarks at the end of § 6.

‡ By this, we mean that an appreciable deflexion in the direction of the magnetic field lines takes place across these shocks.

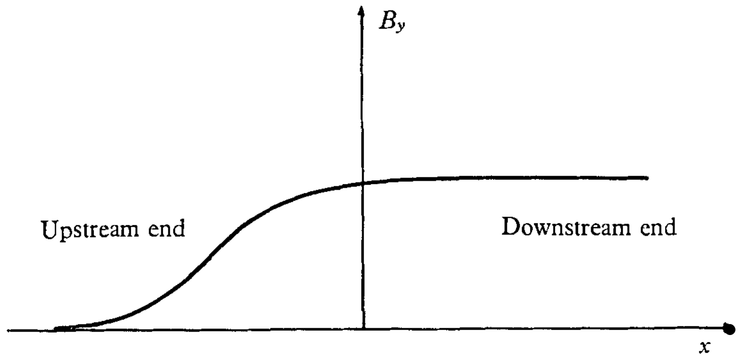

FIGURE 6. The profile of B_y in the undisturbed steady state.

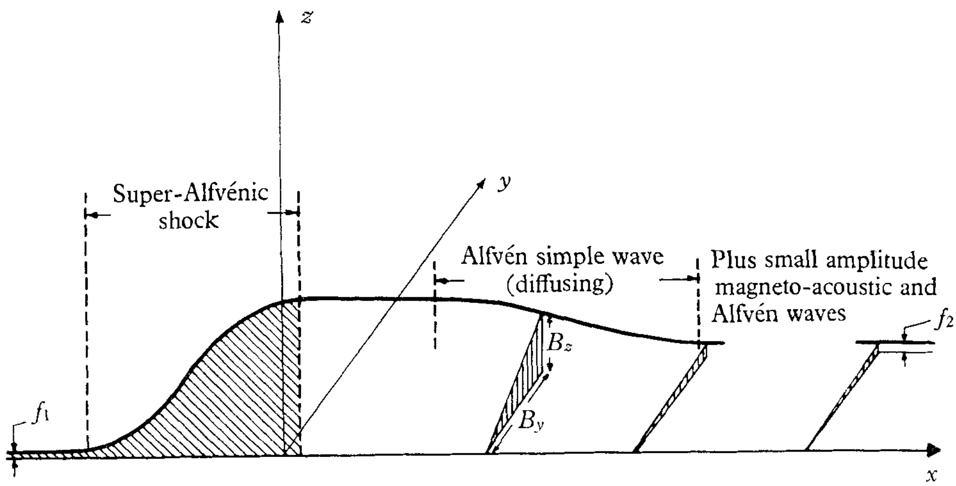

FIGURE 7. The profile of (O, B_y, B_z) in the breakdown configuration. ▧, plane, $y = 0$; ▥, plane, $x = \text{const.}$

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